J. Fluid Mech. (2000), *vol.* 416, *pp.* 187–195. Printed in the United Kingdom

On the application of box models to particle-driven gravity currents

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(Received 30 November 1998 and in revised form 25 February 2000)

New laboratory experiments on different types of lock-exchange particle-driven gravity currents advancing into a flume of fresh water are presented. These include purely saline currents, monodisperse particle-laden gravity currents with both fresh and saline interstitial fluid, and bidisperse particle-laden currents. For each case a simple box model is developed. These agree well with the experimental data. We find that particulate gravity currents with saline interstitial fluid flowing into ambient fresh fluid are best described using a Froude number of 0.52 in the box model (cf. Huppert & Simpson 1980). However, particulate gravity currents with fresh interstitial fluid are best described using a higher Froude number of 0.67. The change in Froude number reflects the different shape and structure associated with the different density of interstitial fluid. For all experiments, box models provide accurate predictions for up to twenty lock-lengths.

1. Introduction

Inertial lock-exchange gravity currents have an initial slumping phase characterized by constant velocity, followed by a self-similar regime during which the length increases at a rate proportional to $t^{2/3}$ (Huppert & Simpson 1980). The transition in flow regime occurs after waves caused by lifting the gate have propagated from the rear of the lock to the nose of the current. Experiments and numerical calculations suggest this takes place once the current has advanced to about ten lock-lengths from the backwall (Rottman & Simpson 1983).

Recent theoretical interest in particle-driven gravity currents has focused largely on the development of box models and two-layer shallow water numerical models to describe the dynamics and deposits of fresh-water monodisperse (single particle size) and bidisperse (two particle sizes) gravity currents. These models follow from the original box model of Huppert & Simpson (1980) which described the slumping phase of a purely saline current. The models have been successfully compared with experimental data from monodisperse gravity currents, suggesting that box models provide an accurate leading-order description of the flow (Dade & Huppert 1995; Bonnecaze, Huppert & Lister 1993; Bonnecaze *et al.* 1995; Hallworth, Hogg & Huppert 1998). The purpose of this note is to show that such box models can accurately replicate propagation measurements of experimental fresh bidisperse and saline monodisperse particle-laden gravity currents in addition to pure saline and fresh monodisperse gravity currents. New experiments are compared with a series of new box models which focus on the slumping phase of such currents. Since the buoyancy of a particle-laden current evolves with distance through sedimentation, the speed of waves propagating from the rear of the lock to the nose of the current may also evolve relative to a purely saline current. The experimental data are therefore also used to test the range of validity of the model for a slumping particulate current.

2. Experiments

2.1. *Method*

New laboratory experiments to examine the propagation of gravity currents were conducted in a tank which was 600 cm long, 20 cm wide and filled to a depth of 40 cm. A gate placed 20 cm from one end of the tank retained the gravity current fluid until the start of each experiment when the gate was removed. Four types of experimental gravity currents were analysed in which the source fluid was (i) pure saline, (ii) saline monodisperse, (iii) fresh monodisperse, and (iv) fresh bidisperse. All currents flowed into a reservoir of fresh water and the position of the current nose was measured every 3 s.

For the first series of experiments, four saline gravity currents were studied using initial masses of salt of 90 g, 180 g, 270 g and 360 g. These give an initial reduced gravity, g', of $4.9\,\mathrm{cm\,s^{-2}}$, $8.8\,\mathrm{cm\,s^{-2}}$, $12.5\,\mathrm{cm\,s^{-2}}$ and $16.3\,\mathrm{cm\,s^{-2}}$ where $g' = g(\rho_a - \rho_c)/\rho_a$. Here, g is the acceleration due to gravity, ρ_a and ρ_c are the densities of the ambient fluid and gravity current respectively.

For the second series of experiments, five saline monodisperse gravity currents were studied using mixtures of salt and 88 μm silicon carbide particles. The total mass of salt and particles employed in each experiment was 180 g and the relative initial proportions of salt and particles were varied. Six experiments were conducted using 100% particles by mass, and 80%, 60%, 40%, 20% and 100% salt by mass.

In the third series of experiments, six fresh monodisperse gravity currents were studied, four using silicon carbide particles and two using soda-lime glass ballotini particles. The average grain sizes of silicon carbide were 25 μm, 69 μm, 88 μm and 105 μm and in each experiment 180 g of sediment was employed, corresponding to $g' = 7.6\,\mathrm{cm\,s^{-2}}$. The grain sizes of ballotini were 54 μm and 84 μm; 750 g was used, giving $g' = 27.2\,\mathrm{cm\,s^{-2}}$.

The fourth series of experiments investigate bidisperse gravity currents, and was conducted by Gladstone, Phillips & Sparks (1998). We present four of their experiments, using two grades of silicon carbide with grain sizes 25 μm and 69 μm. In each experiment the initial reduced gravity was $7.6\,\mathrm{cm\,s^{-2}}$. The relative proportions of each grade was varied in the four experiments so that the coarse fraction comprised 20%, 40%, 60% and 80% by mass, with the remainder made up of the fine fraction.

The reproducibility for purely saline gravity currents is in the order of 1.5%. For currents partly or wholly driven by particles this error increases to 2.5%. This is primarily due to sedimentation of some particles in the lock region prior to lifting the lockgate.

2.2. *Results*

The experiments reveal that, in comparison to their purely saline counterparts, the propagation speed of purely particle-driven gravity currents decreases with distance from source because particles are continually falling out of suspension, thereby reducing the density difference between the current and the ambient fluid (figures 1 and 2) (Middleton 1966). Furthermore, currents driven by fine monodispersed particles with low settling speeds travel faster and further than currents driven by coarse monodispersed particles with high settling speeds (figures 3 and 4) (e.g.

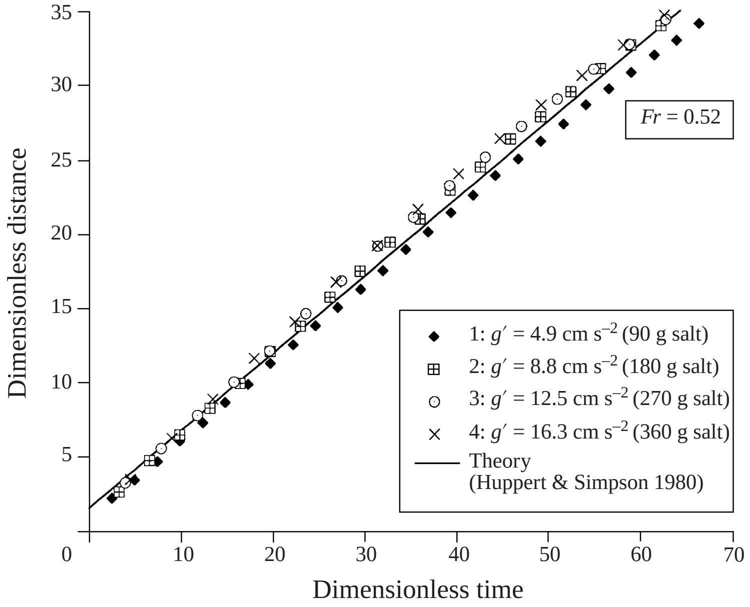

FIGURE 1. Pure saline gravity currents are scaled using the box model expression (4), collapsing to a straight line with a best-fit Froude number of 0.52. This confirms observations of Huppert & Simpson (1980).

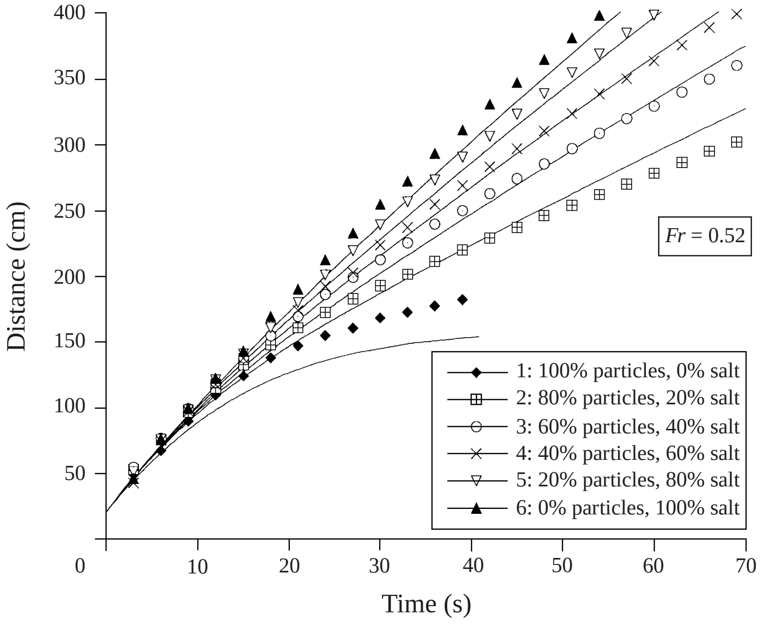

FIGURE 2. Box model predictions using expressions (13) and (14) are fitted to saline monodisperse gravity current data. An optimum Froude number of 0.52 is produced. This is the same as that obtained for pure saline currents.

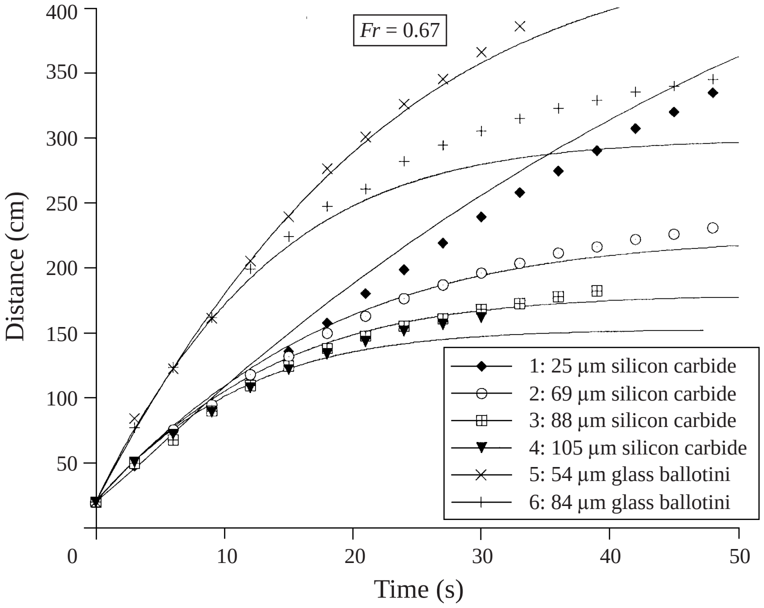

FIGURE 3. Box model predictions using expressions (6) and (7) are fitted to fresh monodisperse experimental data. A Froude number of 0.67 is more appropriate than the value of 0.52 found for saline currents, indicating that monodisperse fresh currents and monodisperse saline currents differ.

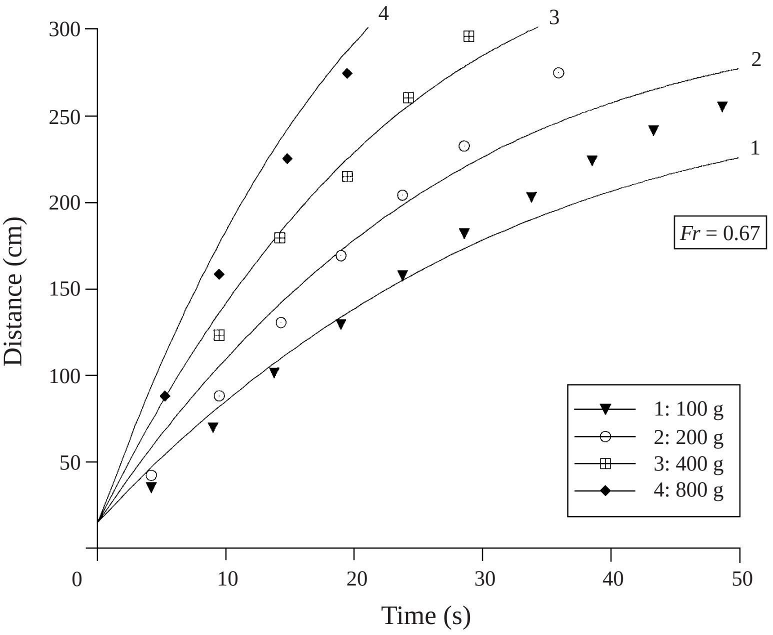

FIGURE 4. Bonnecaze *et al.* (1995) conducted four monodispersed gravity current experiments using varying initial sediment masses of 37 μm silicon carbide. Box model predictions are successfully applied to these experiments using $Fr = 0.67$.

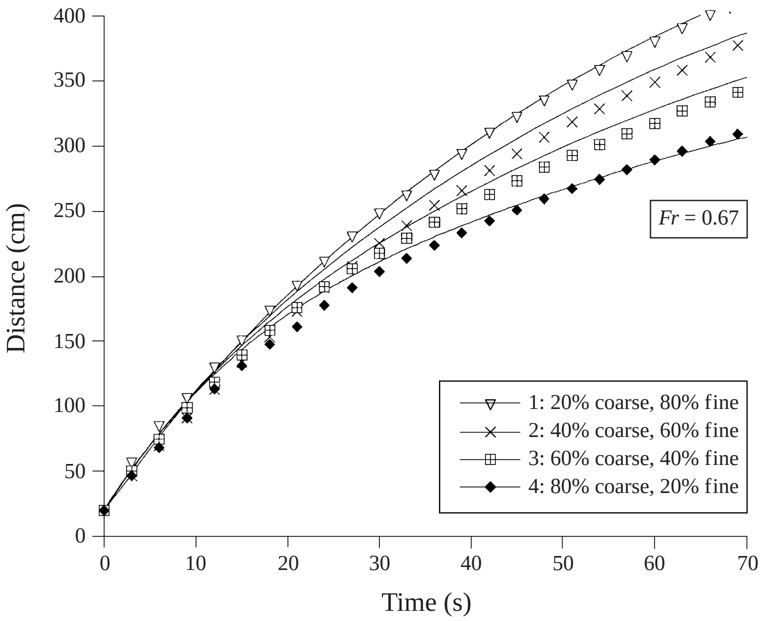

FIGURE 5. Box-model predictions using expressions (9) and (10) are plotted with the bidisperse experimental results of Gladstone *et al.* (1998). These also produce an optimum Froude number of 0.67.

Bonnecaze *et al.* 1993). Bidisperse gravity current data reflect this: currents driven by a high fraction of fine particles travel faster than currents driven by a high fraction of coarse particles (figure 5) (Gladstone *et al.* 1998).

3. Box models

Box-model expressions presented here have been obtained by combining (1) the conservation of volume, (2) a Froude number condition at the flow head (Huppert & Simpson 1980) and (3) an Einstein law of sedimentation for each particle size (Hazen 1904):

$$Q = hl, \tag{1}$$

$$\frac{\mathrm{d}l}{\mathrm{d}t} = Fr\left(\frac{H}{h}\right)^{1/3}(g'_T h)^{1/2}, \tag{2}$$

$$\frac{\mathrm{d}g'}{\mathrm{d}t} = -\frac{v_s g'}{h}, \tag{3}$$

where Q, h and l are the two-dimensional area, height and length of the gravity current respectively, H is the ambient fluid depth, g'_T denotes the total reduced gravity of the current caused by the presence of both particles and salt and g' denotes the reduced gravity of the current associated with the presence of particles of settling speed v_s. Equation (2) is an empirical law established by Huppert & Simpson (1980) which they found to be valid when $h \geqslant 0.075\,H$ with a Froude number of 0.5.

For purely saline gravity currents, (1) and (2) lead to the prediction that l increases with t according to (Huppert & Simpson 1980)

$$l^{7/6} = l_0^{7/6} + Fr\tfrac{7}{6}g_0'^{1/2}H^{1/3}Q^{1/6}t, \tag{4}$$

where the subscript zero denotes initial values.

In fresh monodisperse gravity currents, the fraction of the initial particle content remaining in the flow is given by p, so that

$$g_T' = g_0'p. \tag{5}$$

Equations (1) to (3) lead to the following expressions relating the current length l, particle content p and time t (e.g. Hallworth *et al.* 1998):

$$l^{13/6} = l_0^{13/6} - Fr\frac{26H^{1/3}Q^{7/6}g_0'^{1/2}}{6v_s}(p^{1/2} - 1), \tag{6}$$

$$t = -\frac{Q}{v_s}\int_1^p \frac{1}{pl}\,\mathrm{d}p. \tag{7}$$

In a fresh bidisperse gravity current, p_1 and p_2 denote the coarse and fine particle content as a fraction of the total initial particle content, and v_1 and v_2 denote their respective settling speeds, so that

$$g_T' = g_0'(p_1 + p_2). \tag{8}$$

Equations (1) to (3) now lead to these expressions relating l, p and t

$$l^{13/6} = l_0^{13/6} - Fr\frac{13H^{1/3}Q^{7/6}g_0'^{1/2}}{6v_1}\int_{p_{1(0)}}^{p_1}\frac{(p_1 + p_2)^{1/2}}{p_1}\,\mathrm{d}p_1, \tag{9}$$

$$t = -\frac{Q}{v_1}\int_{p_{1(0)}}^{p_1}\frac{1}{p_1 l}\,\mathrm{d}p, \tag{10}$$

where

$$p_2 = p_{2(0)}\left(\frac{p_1}{p_{1(0)}}\right)^{v_1/v_2}. \tag{11}$$

Finally, for gravity currents driven by both salt and particles, where a denotes the fraction of the initial buoyancy due to the salt and b denotes the fraction of the initial buoyancy due to the particles,

$$g_T' = g_0'(a + b). \tag{12}$$

From (1) to (3), the length versus time relationship is given by

$$l^{13/6} = l_0^{13/6} - Fr\frac{13H^{1/3}Q^{7/6}g_0'^{1/2}}{6v_s}\int_{b_0}^{b}\frac{(a + b)^{1/2}}{b}\,\mathrm{d}b, \tag{13}$$

$$t = -\frac{Q}{v_s}\int_{b_0}^{b}\frac{1}{bl}\,\mathrm{d}b. \tag{14}$$

4. Comparison to experiments

We now compare these expressions with a variety of new and previously published experimental data (Bonnecaze *et al.* 1993; Gladstone *et al.* 1998). Since expression (2) is empirical, we use the new experimental data to determine the optimum value for

Fr. First the model of a purely saline current is compared with the new experimental data (figure 1). We find that $Fr = 0.52$ gives the best fit for these currents, in good agreement with observations of Huppert & Simpson (1980). The experimental data collapse well. There is a weak dependence on g', although this trend is of the same order as experimental errors.

This value of $Fr = 0.52$ also corresponds to the best-fit value for gravity currents driven by particles suspended in saline interstitial fluid (figure 2). The model is in good agreement with our experimental observations: theoretical predictions typically lie within 4% of experimental data (experiments 1–4; figure 2), although in one case this deviates by up to 7% (experiment 5; figure 2). With a monodisperse current containing no salt, using a Froude number of 0.52 produces predictions which deviate by nearly 20% from experimental data (experiment 6; figure 2).

However, for particle-driven gravity currents in which the interstitial fluid has the same density as the ambient fluid, we find that the optimum Froude number to fit the experimental data has value $Fr = 0.67$. This same value emerges for both monodisperse and bidisperse particle-driven gravity currents in which the interstitial fluid has the same density as the ambient fluid (figures 3, 4 and 5). These comparisons indicate that fresh particle-driven gravity currents differ from saline particle-laden currents propagating through fresh ambient fluid.

Although the box model of Huppert & Simpson (1980) is only valid while $h \geqslant 0.075\,H$, corresponding to a distance of 2.67 m in our flume, we find that theoretical predictions are accurate to a distance of approximately 4 m. Beyond this, our experimental currents travel faster and further than the theory predicts. This may be due to the range of grain sizes present in each distribution. As larger particles settle out, the effective fall velocity of the remaining particles will decrease (Martin & Nokes 1989). This is not included in the box model.

5. Discussion

The interesting feature of these experiments and corresponding box model predictions is that the coefficient in the Froude relation (2) is nearly 30% greater for fresh particle-laden gravity currents than for saline particle-laden gravity currents. This disparity in Froude number suggests that there is a fundamental difference between these two types of current, caused by the different densities of the interstitial fluid. A difference between the dynamics of fresh particulate currents and pure saline currents has been noted by Bonnecaze *et al.* (1995) and Hallworth *et al.* (1998). They found that when using the Froude number of Huppert & Simpson (1980), the box model overestimates the run-out distance of fresh particulate currents and suggested inclusion of a multiplication factor of 1.6. This same factor was used to scale time by Hogg, Huppert & Hallworth (1999) in box model analysis of buoyant lift-off currents. The experiments and box models presented herein suggest that particle-laden gravity currents with neutrally buoyant interstitial fluid, may in fact be very accurately described using a Froude number of 0.67.

Visualization of particle-laden currents suspended in fresh or saline interstitial fluid indicates that there is a difference in current shape depending on the interstitial fluid (figure 6). Both currents in these photographs contain 53 μm silicon carbide particles and are driven by a bulk initial reduced gravity $g_o' = 8\,\mathrm{cm\,s^{-2}}$. They are flowing into fresh ambient water. Figure 6(*a*) is a fresh particle-driven current. Figure 6(*b*) is a saline particle-driven current.

Comparison between figures 6(*a*) and 6(*b*) indicates that the current driven by

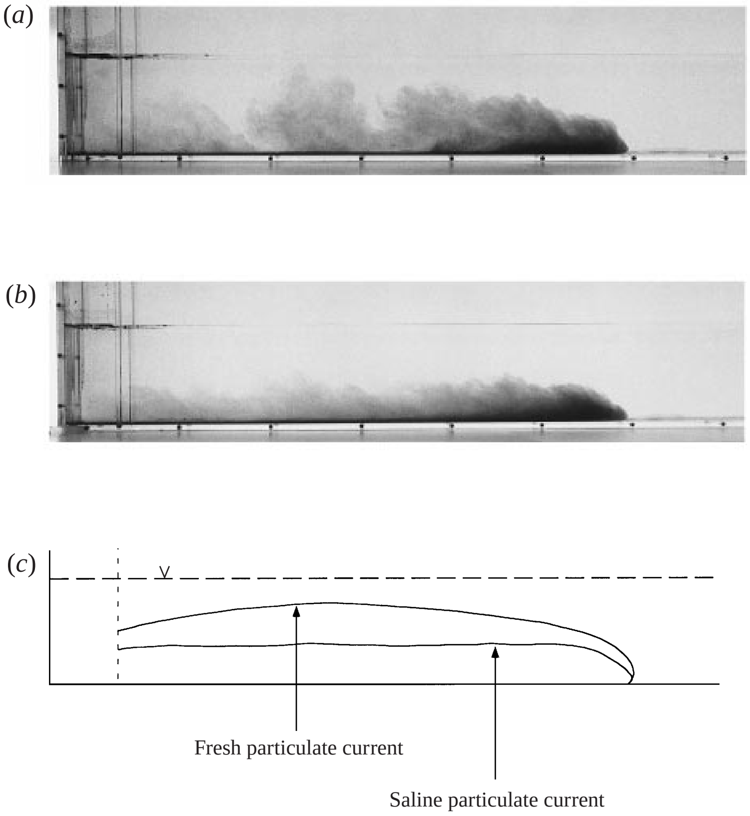

FIGURE 6. Visualization of two particle-laden gravity currents with (*a*) fresh interstitial fluid and (*b*) saline interstitial fluid. In (*c*) we have superimposed the average profile for each current from the photographs to illustrate the difference in current shape.

particles suspended in fresh interstitial fluid has deepened and developed a substantial turbulent wake region along its upper edge, in contrast to the current driven by particles suspended in saline fluid. As particles sediment from a fresh gravity current, the bulk density decreases towards that of the overlying fluid. The current can therefore deepen with distance and time as particles are lofted upwards, and mixing of current fluid and ambient fluid occurs (figure 6*a*). In contrast, the residual fluid in a particulate current with saline interstitial fluid remains more dense than the purely fresh ambient fluid during sedimentation. As a consequence of this density contrast between interstitial and ambient fluids, mixing of current fluid and ambient fluid is suppressed and these saline currents do not deepen as much as those where the interstitial and ambient fluids are of equal density (figure 6*b*). This leads to a difference in shape and structure between these two currents, as summarized in figure 6(*c*), and manifested in the two different Froude numbers.

Our experiments have identified the behaviour of the two end member particle-laden flows in which the density of the interstitial fluid is either (i) the same or (ii) denser than the ambient fluid. We anticipate that there is a smooth transition of Froude number from 0.67 to 0.52 as the density of the interstitial fluid of a particle-laden gravity current increases from that of the ambient fluid. In this intermediate regime, the Froude number may also evolve with distance as the current sediment load evolves. Investigation of the details of this transition could form the subject of further study.

In summary, the good agreement between experimental results and theoretical predictions shows that box models can provide an accurate description of particle-laden laboratory gravity currents.

C.G. is funded by EPSRC.

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